Shock Reflection Transition in Three-Dimensional Steady Flow About Interfering Bodies

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A numerical study of the three-dimensional flowfield produced by the interaction of an axisymmetric shock and a planar wall is presented. These types of flowfields are encountered when a store is in close proximity to an aircraft. The flow is assumed inviscid and everywhere supersonic. This work is based upon the numerical solution of Euler's equations with all shock waves fit as discontinuities. The axisymmetric shock reflects from the plate first in a regular fashion, but, ultimately, this reflection becomes a Mach reflection. The transition from regular to Mach reflection and the subsequent development of a cross-sectional Mach disk is computed explicitly and studied in detail. The computational procedure is outlined and numerical results are compared with experimental data.

Introduction

THE study of multiple interfering bodies in high speed flow is important to the development of a number of vehicles. This paper is concerned with the types of interference flowfields that are encountered when a supersonic aircraft carries stores externally. The question of the feasibility of store external carriage for a supersonic cruise aircraft is an important one. Internal store carriage has the penalties of increasing aircraft volume and store separation problems, while external carriage has the penalties of increased drag with somewhat alleviated separation difficulties. In order to achieve safe store separation, detailed knowledge of the forces and moments on the store are required during the entire separation process.

The present effort is intended to study the basic fluid mechanics of a store in close proximity to an aircraft, thereby gaining insight into the feasibility of external store carriage and separation at supersonic speeds. Specifically this paper deals with the reflection of a shock, produced by an axisymmetric store, from a planar wall which represents the underside of an aircraft or wing. The configuration is as shown in Fig. 1. No reflections of the shock from the store will be considered in the present paper.

Current methodology requires extensive wind-tunnel testing of specific aircraft store combinations. In order to cost effectively evaluate aerodynamic concepts involving store shape and aircraft concavity tailoring, accurate and efficient computational tools need to be developed. The flowfield adjusts so quickly during separation that a steady-state computation at each instant of the process is assumed to predict accurately the forces on the store, the unsteady fluid mechanics being negligible. The viscous effects at the Mach numbers and Reynolds numbers considered here are confined to the boundary layers on the store and wall. The present paper outlines a numerical scheme for the prediction of the inviscid supersonic flow about a simple store in close proximity to a reflection plane. In addition, the prediction procedure is used to study the basic fluid mechanic issues inherent in complex shock interactions, particularly the transition of the reflection and the subsequent development of a cross-sectional Mach stem (Fig. 2).

Presented as Paper 82-0306 at the AIAA 20th Aerospace Sciences Meeting, Orlando, Fla., Jan. 11-14, 1982; submitted Jan. 22, 1982; revision received Sept. 7, 1982. Copyright American Institute of Aeronautics and Astronautics, Inc., 1982. All rights reserved.

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The calculation of the point where the axisymmetric shock and the plate meet in each cross plane ($z=\text{const}$) is two-dimensional in a plane normal to the curve A of Fig. 1. This curve is made up by the intersection of a well-defined axisymmetric shock and a plate, and it is a hyperbola if the store is a circular cone. The incident or axisymmetric shock turns the flow toward the plate and the reflected shock has the sole purpose of turning the flow back parallel to the plate (Fig. 2a). The component of the freestream Mach number in the local plane normal to the curve A is decreasing continually as the flow proceeds downstream. Due to this decrease in relative Mach number, at some point the reflected shock can no longer turn the flow parallel to the plate, resulting in a Mach disk (Fig. 2b). The basic mechanism is the same as in

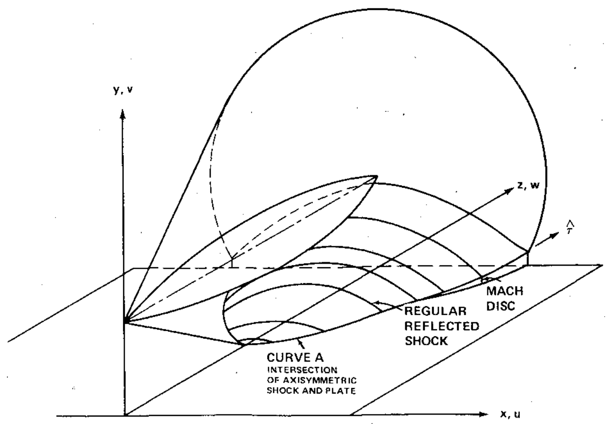

Fig. 1 Sketch of store/plate configuration with reflecting shock.

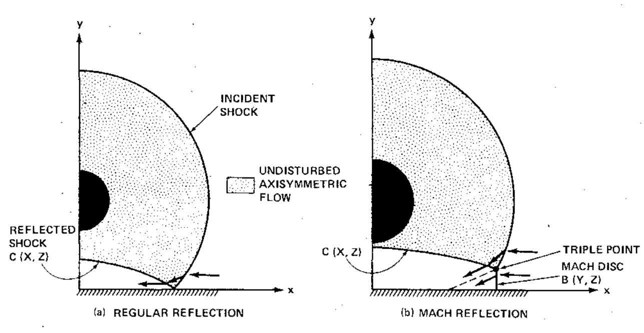

Fig. 2 Cross-sectional regular and Mach reflection.

two-dimensional flow and occurs in many shock interaction problems. Previous investigators have found anomalies between existing theories and experiments associated with the transition phenomenon, and a summary of these findings is presented in a review paper by Griffith.[1]

Calculations that rely on the small perturbation assumption (linearized theory) may be misleading since the fluid mechanics of a store in close proximity to an aircraft is dominated by the reflection of a shock system that requires a detailed nonlinear calculation for reliable prediction. For this reason solutions to Euler's equations are sought in the present work. The possibility of computing the complex shock system of the flow, using no special provisions (i.e., "capturing" the shocks) was considered early in this work, but because of the importance of an accurate prediction of these shocks, this possibility was rejected. All of the shock capturing schemes tested resulted in shock waves that were spread over six or more mesh intervals and had oscillations on the high or low pressure side of the shock. The numerical procedure used in the present work employs a finite difference marching technique to integrate Euler equations. All shock waves in the flowfield are fit and careful consideration is given to the cross-sectional triple point (Figs. 1 and 2). The contact surface generated at the triple point is fit in an approximate fashion.

In this paper first the computational procedure used will be outlined, with particular attention paid to the new features of the calculation. A detailed study of the transition of the shock reflection from regular to Mach will follow. A number of sample store/plate flowfield calculations will be presented including a comparison with experimental data. Finally, the conclusions arrived at in this work will be summarized briefly.

Computational Technique

The computational procedure used in this work is based upon a finite difference marching technique. The marching coordinate is on the plate and in the direction of the cone axis (z direction of Fig. 1). Initial conditions are given in the x-y plane and the calculation proceeds in the z direction by integrating Euler's equations. The coordinate system used is shown in Fig. 1, and the coordinates x,y,z are nondimensionalized with respect to the separation distance (i.e., the vertical distance between the plate and the store axis). All the stores considered have a sharp cone at their nose and an axis parallel to the plate so that the store is at zero angle of attack. The freestream velocity vector is aligned with the plate. Under these restrictions the initial conditions can be computed as a Taylor-Maccoll solution. The marching solution proceeds in the z direction with the step size Δz satisfying the C.F.L. condition for stability. The component of velocity w in the z direction must be everywhere supersonic in order for the marching scheme to continue.

The flow on the low-pressure side of the reflected shock (shaded area of Figs. 2a and 2b) remains axisymmetric. This region is computed with an axisymmetric marching procedure. The step sizes for this calculation and that in the fully three-dimensional region under the reflected shock are set equal so that the low-pressure side of the reflected shock is simply evaluated. A computational grid is developed by normalizing the vertical coordinate y so that $Y=y/C(X,Z)$ and the horizontal coordinate x so that $X=x/B(Y,Z)$. The marching direction z is preserved so that $Z=z$. In the computational plane the region under the reflected shock is a rectangle in each plane $Z=$const. The coordinate transformation from x,y,z to X,Y,Z is singular at the regular reflection point (the point where the incident shock hits the plate, Fig. 2a) before the Mach disk develops since $C(1,Z)=0$. The computation of the regular reflection point does not need the evaluation of this transformation or its derivatives so this singularity causes no difficulties.

The finite difference scheme used is the characteristic based scheme presented by Moretti.[2] The scheme is based on the concept of evaluating the spatial differences in the X and Y direction depending on the slope of the bicharacteristics in the X-Z and Y-Z planes, respectively. This scheme is stable in the presence of captured shocks and contact surfaces and closely relates the physical and numerical domains of dependence of each grid point. The boundary condition at the plate ($y=0$, Fig. 2) requires that the velocity normal to the plate vanishes. This boundary condition is satisfied by using a scheme proposed by Abbett.[3] It involves correcting the velocity predicted by the finite difference scheme to make it tangent to the wall via a Prandtl-Meyer expansion or compression. Symmetry is imposed along $x=0$ (Fig. 2).

The incident shock is computed along with the axisymmetric flowfield. The computational procedures used for this part of the flowfield are very similar to those used for the three-dimensional portion of the flow. The reflected shock $C(X,Z)$ (Figs. 2a and 2b) is computed using a scheme developed by Rudman[4] and independently by de Neef.[5] The scheme is based on the concept of correcting the independent variables computed via the finite difference scheme at the shock. The correction is evaluated by simultaneously satisfying the Rankine-Hugoniot relations across the shock and the compatibility relation on a bicharacteristic reaching the shock on its high-pressure side. In the present work the characteristic in the Y-Z plane is used. The regular reflection point, the point where the axisymmetric shock hits the plate, is computed by turning the flow behind the incident shock (Fig. 2a) tangent to the plate, thereby satisfying the plate boundary condition. This computation is performed in a plane normal to the curve A (Fig. 1), which is computed from the intersection of the axisymmetric shock and the plate. This is the only proper plane in which to evaluate the regular reflection point since it contains the normal to the incident and reflected shocks. As this calculation proceeds downstream, the Mach number component in this plane decreases because the curve A (Fig. 1) becomes more swept with respect to the freestream velocity vector. The deflection through which the reflected shock must turn the flow remains relatively constant. The reflected shock sees a decreasing Mach number, and so at some point there is no reflected shock which can turn the flow parallel to the plate. This will be referred to as the detachment point since the phenomenon is similar to the situation at the leading edge of a wedge when the incident Mach number drops below a certain value and the attached shock detaches. In the present work the detachment point has been found to be the point of transition from a regular reflection (Fig. 2a) to a Mach reflection (Fig. 2b). There have been other criteria proposed for the transition point, but the present work indicates that detachment is the only workable criterion for the problem considered in this present paper. This finding will be discussed in more detail in the following section of this paper.

Once the Mach disk configuration of Fig. 2b is developed, the Mach stem $B(Y,Z)$ is computed in the same manner as discussed earlier for the reflected shock $C(X,Z)$. The only difference is that the bicharacteristic used in the Mach stem computation is the one in the X-Z plane. The triple point is computed using a scheme developed by the author for conical corner flows.[6] If we define a unit vector $\hat{\tau}$ (Fig. 1) (having components τ_x, τ_y, and τ_z) to be directed along the locus of all the cross-sectional triple points, only two of its components are unknown since it is a unit vector $\tau_x^2+\tau_y^2+\tau_z^2=1$. Since the triple point in each cross section must be on the known incident (axisymmetric) shock, the tangency condition $\hat{\tau}\cdot\hat{I}=0$ must be satisfied where $\hat{I}$ is the known unit vector normal to the incident shock. The unit vector and tangency conditions leave one component of $\hat{\tau}$ to be determined from the flowfield. The triple point calculation in each cross section proceeds by guessing a value of the z component of $\hat{\tau}$, $\hat{\tau}_z$. With $\hat{\tau}$ defined, the conditions of the high-pressure side of the triple point can be computed by matching pressure and flow deflection on either side of the contact (the von Neumann conditions) (Fig. 2b). This computation is performed in a plane normal to the guessed vector $\hat{\tau}$. The iteration is continued until the conditions on the reflected shock side of the contact surface satisfy the compatibility condition along the bicharacteristics reaching the triple point in the X-Z plane.

Initially, difficulty was encountered with this calculation. It turned out that the evaluation of a term involving the derivative of the vertical velocity in the vertical direction was very important. The gradients in the x direction are very large as the Mach disk formation is approached, and it is the derivative of the vertical velocity in the y direction that balances these large gradients. This phenomenon is analogous to the conical compression between the shock and surface of a circular cone being balanced by the source term in the axisymmetric continuity equation. The only way the triple point iteration could be made to converge at the transition point was to evaluate this derivative at the new station. Here the Mach disk is infinitesimal and the vertical velocity at the triple point is small but finite, yielding a finite value of the derivative. If one considers the compatibility equation on the bicharacteristic reaching the triple point at transition, it becomes obvious how the large gradients in pressure are balanced by this velocity derivative at transition. It also should be noted that only if the flow behind the regular reflection point is supersonic relative to the reflection point does the bicharacteristic exist. Therefore, only if this relative Mach number is supersonic is the reflection point affected by the flowfield, and it is only the bicharacteristic which can determine the initial motion of the triple point. This will be discussed in more detail in the next section.

The contact surface generated at the triple point is fit in an approximate fashion. Its location in the flowfield is computed by assuming it is straight, the slope being computed at the triple point. Velocity components and entropy are not differenced across the approximate contact location.

For the higher freestream Mach number cases considered in this work, the flow on the reflected shock side of the contact is slightly supersonic relative to the triple point. This situation results in the well-known double Mach reflection (Fig. 3).[7] The second normal shock is caused by the fact that the flow behind the triple point is supersonic and directed toward the wall. In all the cases considered here the Mach number behind the triple point was so slightly supersonic that the deflection required to turn the flow tangent to the wall was beyond detachment, precluding the possibility of an oblique shock and the complex Mach reflection.[7] This situation was encountered near transition when the Mach disk is infinitesimal in a few of the cases studied. The Mach number behind the triple point on the reflected shock side of the contact quickly became subsonic so that the double Mach reflection is impossible to see in any of the results to be presented later in this paper. It must be noted that when the Mach number relative to the triple point is supersonic, no bicharacteristic reaches the triple point on the reflected shock side of the contact. In this case a bicharacteristic is used to compute the high-pressure side of the second Mach stem (Fig. 3), and the supersonic flow is assumed constant relative to the primary triple point. The triple point iteration is slightly modified in that after the flow behind the triple point is computed with an assumed τ_z a normal shock is inserted. Now, the iteration is continued until the characteristic reaching the high-pressure side of the second Mach stem is satisfied. At the low end of Mach number spectrum ($M_\infty < 2.2$), the freestream Mach number relative to the triple point goes below 1.5. For these conditions there is no triple point solution that satisfies the von Neumann conditions (matching of pressure and flow deflection). In this situation the Guderley conditions[8] must be used. Guderley's hypothesis is that at the triple point the reflected shock is sonic and the flow behind it is expanded in order to match the pressure and flow deflection produced by the Mach stem at the triple point. These conditions were applied successfully by the present author[9] for low Mach number conical corner flows. These conditions have not been implemented in the present work, so in this paper only Mach numbers above 2.2 will be considered.

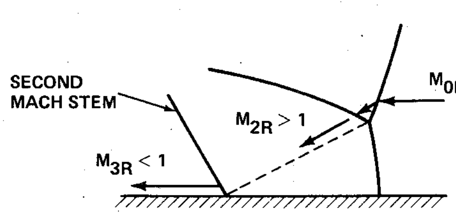

Fig. 3 Double Mach reflection.

Transition from Regular to Mach Reflection

The reflected shock in the regular reflection configuration (Fig. 2a) turns the flow, which has been deflected toward the plate by the incident shock, parallel to the plate. The only correct way of evaluating this process is in a plane normal to the intersection of the incident shock and the plate (curve A of Fig. 1). The physical phenomenon can be understood more clearly by studying the shock polars corresponding to the projected Mach numbers at different steps downstream. For the sake of simplicity let us consider the supersonic flow about a 15-deg half-angle cone in close proximity to a plate. In this case the intersection of the conical shock and the plate forms a hyperbola. The Mach number component in a plane normal to the hyperbola can be computed at each marching station, and the incident and reflected shock polars can be generated. Figure 4 shows a series of these polars. Figure 4a corresponds to the station where the axisymmetric shock first hits the plate. At this station the relative Mach number is the

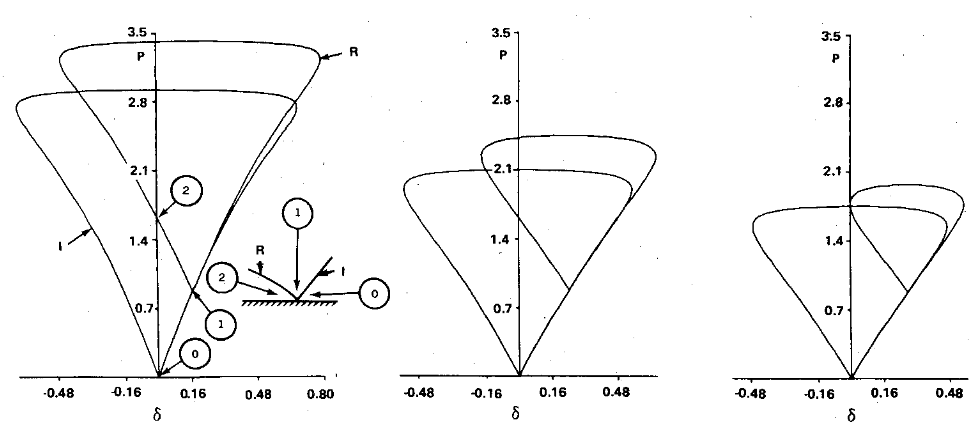

Fig. 4 Shock polars for a regular reflection from the initial interaction to detachment; $M_\infty = 4$, 15-deg half-angle cone.

freestream Mach number ($M_\infty = 4$ in the case shown) since the hyperbola is normal to the freestream velocity when the conical shock first hits the plate. Figure 4 is a plot of pressure $[P = \ln(p/p_\infty)]$ vs flow deflection (δ is measured relative to the plate) for all possible shock orientations. The polar I corresponds to the incident shock and polar R corresponds to the reflected shock. The conditions indicated 0 in Fig. 4a are the freestream conditions; 1 indicates conditions behind the incident shock; 2 indicates the conditions behind the reflected shock. Figure 4b shows shock polars further downstream; note that both the incident and reflected polars have shrunk. This is due to the fact that the relative freestream Mach number has decreased because the hyperbola has become swept relative to the freestream velocity. Figure 4c shows the detachment condition in which the reflected shock is just able to turn the flow tangent to the plate. It is important to note that at this condition the flow behind the reflected shock is subsonic, and therefore characteristics are reaching the reflection point from inside the flowfield. In the present work transition to a Mach disk is started at this detachment station, with the characteristic from inside the flowfield determining the motion of the triple point.

Von Neumann was the first to propose the detachment criterion of transition from a regular to a Mach reflection.[10] In the same paper von Neumann proposed another possibility that he called the "stationary case," which exists only in the higher Mach number regimes. Figure 5a shows the shock polars at which transition should take place if the stationary criterion is used. It should be noted that since von Neumann's work in this area, Henderson and Lozzi[11,12] have conducted many experiments in two-dimensional steady and pseudosteady flows and concluded that for high Mach number flows the stationary criterion (they refer to the condition as "mechanical equilibrium") is correct. Henderson and Lozzi reasoned that there is no wave system that could balance a jump in pressure at transition. They concluded that transition occurs when the pressure and flow deflection at the regular reflection point matches those which occur at the triple point (Fig. 5a). The present work shows how the jumps in pressure and flow deflection balance each other when detachment is used as the transition criterion. Ben-Dor et al.[13,14] have conducted experiments in fully unsteady flows that indicate that none of the existing criteria for transition matched the data they obtained. A hysteresis loop was found in the regular to Mach and Mach to regular transition phenomenon in the experiments of Ref. 13. To add to the controversy a number of experiments have indicated "persistent regular reflections," that is, regular reflections beyond detachment where they can not theoretically exist (for example, Ref. 15). There are a number of serious difficulties in trying to determine transition experimentally, in particular shock wave/boundary-layer interaction in the experiments in which the shock is reflected from a solid wall. In addition, downstream boundary conditions and tunnel blockage can prematurely force a Mach disk.[12] The only experiments that studied continuous transitions were those of Refs. 13 and 14 for the unsteady flow of a blast wave hitting a concave and convex model. Shock wave/boundary-layer interaction is important in these experiments. No detailed experiments on transition have been conducted for the flowfield considered here, although the problem considered here would be a good candidate for the study of transition from regular to Mach reflection.

There exists one other transition criterion: the sonic condition (Fig. 5b). This condition is very close to detachment as can be seen by comparing Figs. 5b and 5c. It would seem very difficult to distinguish the two either experimentally or numerically. Henderson performed experiments on the reflection of weak waves in pseudosteady flow[15] which indicated that for weak waves the sonic condition is correct. The sonic condition predicts transition as soon as the flow behind the regular reflection is sonic (Fig. 5b) relative to the reflection point.

All of the currently popular transition criteria are based on purely inviscid arguments. The possibility that the transition to and subsequent development of the Mach stem is dominated by viscosity has been investigated by other researchers, particularly in the weak shock regime.[16] Most of the experimental evidence currently available indicates that the transition phenomenon is independent of Reynolds number. Additionally, the work presented in Ref. 14 indicates that the shock reflection transition is independent of surface roughness. Thus far there is no hard evidence that transition is dominated by viscosity, but it is impossible to discount these effects, particularly in light of the discrepancies between experimental results and the inviscid theories in unsteady and pseudosteady flows. The current study is an inviscid analytical/computational one so that all conclusions are based on inviscid arguments.

In the present work all three transition criteria were studied. Only by using the detachment criterion could a solution be found for the triple point motion at the first few steps after transition. Both the mechanical equilibrium ("stationary") and sonic conditions occur before detachment when the flow behind the regular reflection point is still supersonic relative to the reflection point. Therefore, there is no flowfield information reaching the reflection point, leaving no additional condition to determine the motion of the triple point up the incident shock. If one looks at this problem as the supersonic flow over a wedge, the reflected shock corresponds to the wedge shock. The reflected shock exists solely to turn a supersonic flow tangent to a solid surface as does the shock at the leading edge of the wedge. It is well known that the wedge shock will remain attached until the detachment condition is reached (barring any back pressure effects). Continuing this analogy, the regular reflection will be maintained until

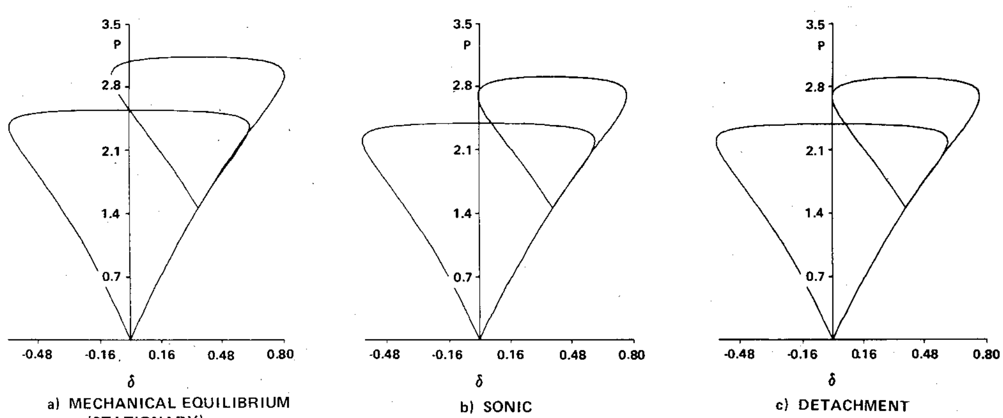

Fig. 5 Transition criterion shock polars; $M_\infty = 6$, 15-deg cone.

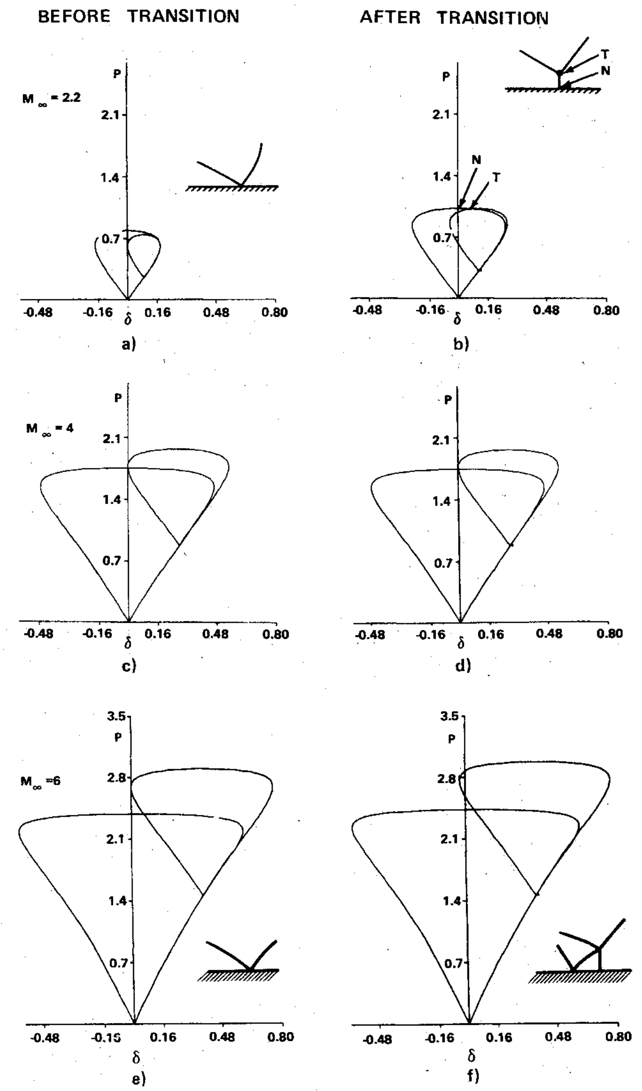

Fig. 6 Shock polars before and after transition; 15-deg half-angle cone.

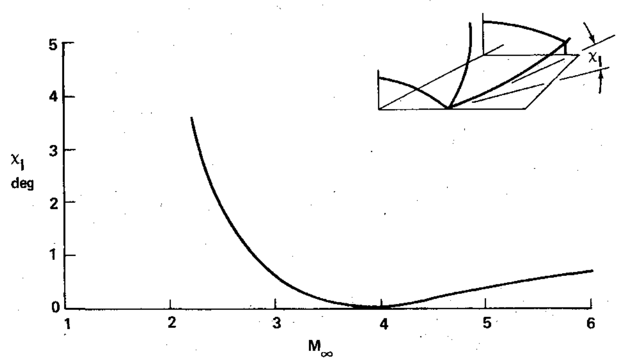

Fig. 7 Initial triple point slope; 15-deg cone.

detachment is reached. This analogy was first used by von Neumann.[10]

Once detachment is reached and transition occurs, the triple point in general does not stay on the plate; it lifts off. Therefore, the Mach number relative to the triple is not the same as that relative to the corresponding regular reflection point. Figure 6 shows this transition in terms of the incident and reflected shock polars before and after transition. In all the cases shown the store is simply a 15-deg half-angle cone. The lowest Mach number shown is 2.2 since this is the lowest Mach number (for a 15-deg cone) that has a von Neumann

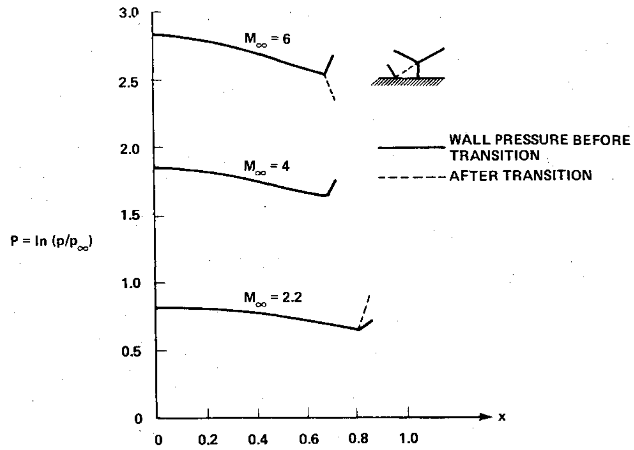

Fig. 8 Wall pressure distribution; 15-deg cone.

triple point solution at transition. This was discussed in the previous section of this paper. Figures 6a and 6b indicate the large change in the relative Mach number at transition and the corresponding change in the conditions at the reflection point. The point T of Fig. 6b marks the triple point condition while N indicates the condition at the base of the Mach stem. Figures 6c and 6d show the polars for a special case ($M_\infty = 4$). In this case the triple point slope remains virtually tangent to the plate at transition; consequently, the polars of Figs. 6c and 6d are virtually identical. It should be noted that this corresponds to the situation in which the mechanical equilibrium criterion and the detachment criterion are just about the same. In addition, the flow behind the triple point is virtually tangent to the plate. Figures 6e and 6f show the Mach 6 case in which the polars do change but not as much as in the Mach 2.2 case. It should be noted that in Figs. 6b and 6f the reflected shock polars intersect the pressure axis (i.e., the flow could be turned back tangent to the wall by the reflected shock). This seems to imply that a regular reflection is still possible. It must be clearly understood that the polars of Figs. 6b and 6f are constructed assuming that the reflection point has moved off the plate. This fact implies the existence of a Mach stem and, therefore, the triple point solution (T of Fig. 6b) is the only possible condition at the reflection point.

Figure 7 shows the initial liftoff slope of the triple point as a function of freestream Mach number for the 15-deg cone. Figure 7 explains the results of Fig. 6 by showing that the triple point lifts off the plate with a larger angle at the low Mach numbers and initially is tangent to the plate near Mach 4 ($\chi_I = 0$ at $M_\infty = 3.87$); also, the slope increases again slightly at the higher Mach number end. It is interesting to note that the angle, for the conditions shown, never exceeds 4 deg. This result would indicate difficulty in experimentally determining the transition point.

Figure 8 shows the plate pressure distribution from the symmetry plane ($x = 0$) to the reflection point a step before and after transition for the three Mach numbers of Fig. 6 (again the store is a 15-deg cone). The figure shows a rapid drop in pressure (looking in the flow direction right to left) just after the reflection point at the station before transition for all three Mach numbers. This is because the pressure at the regular reflection point rises as detachment is approached. After detachment the pressure distribution at $M_\infty = 6$ exhibits a compression after the base of the Mach stem. This is due to the second shock typical of the double Mach reflection configuration discussed in the previous section of this paper. The $M_\infty = 4$ pressure distribution does not change at all after transition because the triple point remains tangent to the plate as noted previously. The expansion after the regular reflection is increased in the $M_\infty = 2.2$ case the step after transition. These waves, the expansion for $M_\infty = 2.2$, the compression for $M_\infty = 6$, and even the expansion in the $M_\infty = 4$, remain

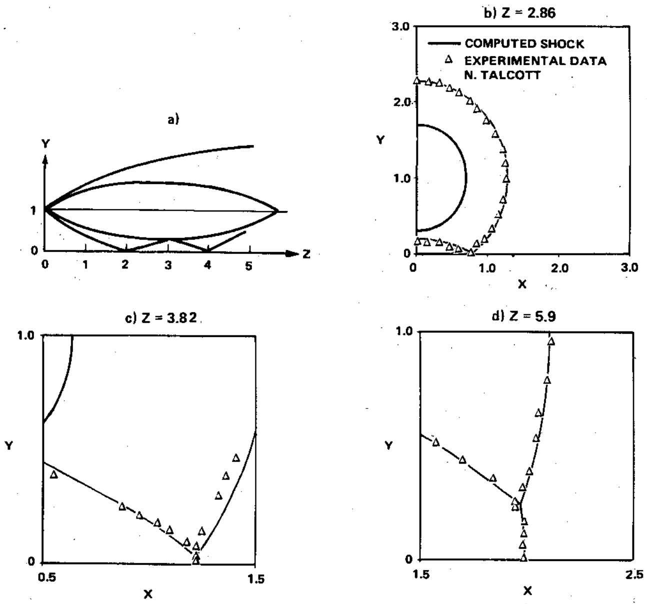

Fig. 9 Comparison of numerical results and experimental data; Sears-Haack body fineness ratio 4.04, $M_\infty = 6.0$.

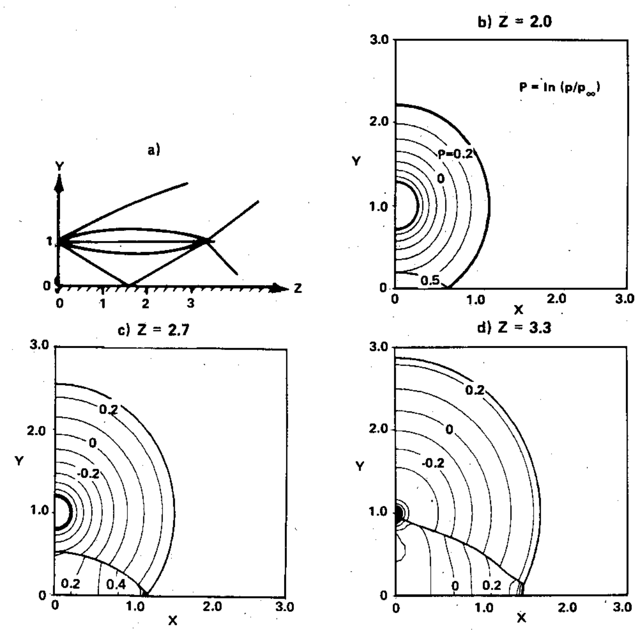

Fig. 10 Cross-sectional shock shape and isobars for a Sears-Haack body; $L/D = 5.5$, $M_\infty = 2.5$.

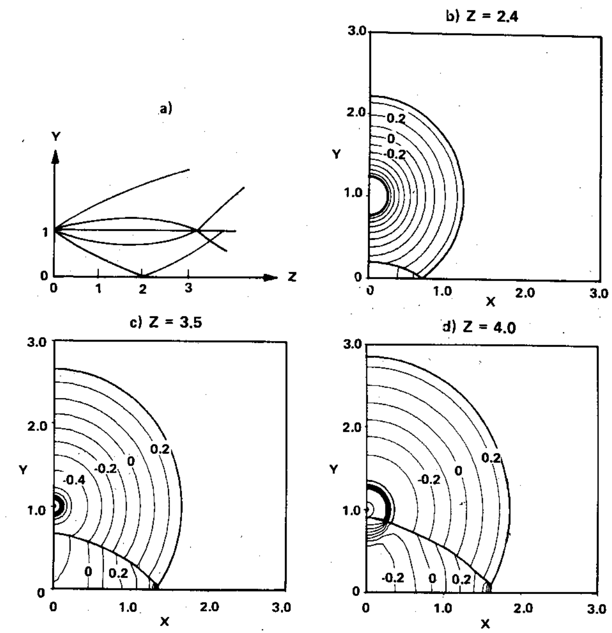

Fig. 11 Cross-sectional shock shape and isobars for a Sears-Haack body; $L/D = 5.5$, $M_\infty = 3$.

close to the triple point and Mach disk and do not move into the flowfield. These are local effects which are balanced by the derivative of the vertical velocity in the vertical direction. As the computation proceeds, the large pressure gradients near the Mach disk decrease in all the cases and eventually disappear. The Mach number on the reflected shock side of the contact in the $M_\infty = 6$ case becomes subsonic relative to the triple point and the second Mach disk is eliminated.

Sample Calculations

In this section the results of a number of store/plate flowfield computations will be presented. In all of these computations only the reflection of the store shock from the plate is studied, that is, the intersection of the reflected shock and the store is not considered. The first computational results to be considered are shown in Fig. 9. The store is a Sears-Haack body with fineness ratio 4.04 (Fig. 9a). A cone is used near the nose of the store, which is true of all the stores studied here. The freestream Mach number is 6 and the ratio of specific heats is 1.4. Figures 9b-d show the computed shock configurations and the experimental results of Talcott.[17] Figure 9b shows the Mach disk configuration just after transition. The Mach disk is too small to be seen either in the computed results or in the experimental vapor screen pictures. In Figs. 9c and 9d the region near the Mach disk is expanded for clarity. The comparison is very good, particularly at the last station shown. This comparison indicates the soundness of the overall computational procedure, and, in particular, it indicates that using detachment as the transition criterion for this type of shock interaction is correct, although more detailed experiments should be run to study this point. If the transition criterion was incorrect, it would seem that the Mach disk heights of Figs. 9c and 9d would not compare with the experiments as well as they do.

Figure 10 shows the results of a computation of the flow about another Sears-Haack store (fineness ratio 5.5, Fig. 10a) at $M_\infty = 2.5$. This is close to the Mach number regime at which store separation for fighter aircraft is important. Figure 10 shows cross-sectional shock shapes and isobars for a number of stations. Figure 10b shows a station just before transition to a Mach reflection, and Fig. 10c shows a station further downstream. The isobars closely spaced near the Mach disk represent a rapid expansion of the flow as was discussed in the previous section. Figure 10d shows a station just after the end of the store. The isobars clustered around the store axis ($y = 1$, $x = 0$) are the shock from the trailing edge of the store. This trailing-edge shock was captured using the λ-Scheme. The size of Mach disk is quite large considering the fact that it has been developing for such a short distance, indicating a rapid growth.

Figures 11 and 12 show results for the same Sears-Haack body (fineness ratio 5.5) at $M_\infty = 3$ and 4, respectively. Figure 11b is a station just before transition and Fig. 11d is past the end of store. The isobars near the disk in Fig. 11d are the expansion after the disk. In this case the interaction of the captured trailing-edge shock and the reflected shock can be seen more clearly. The small dot at the store axis ($y = 1$, $x = 0$) is a very small sting placed at the end of the store in all the cases shown. The isobars representing the trailing-edge shock are more closely spaced in the axisymmetric region of the flow than below the reflected shock. The computational resolution in this region is better than that in the three-dimensional region, 100 radial points. In all the cases shown the resolution below the reflected shock was 10 points vertically and 25 points horizontally. A number of grid convergence tests were

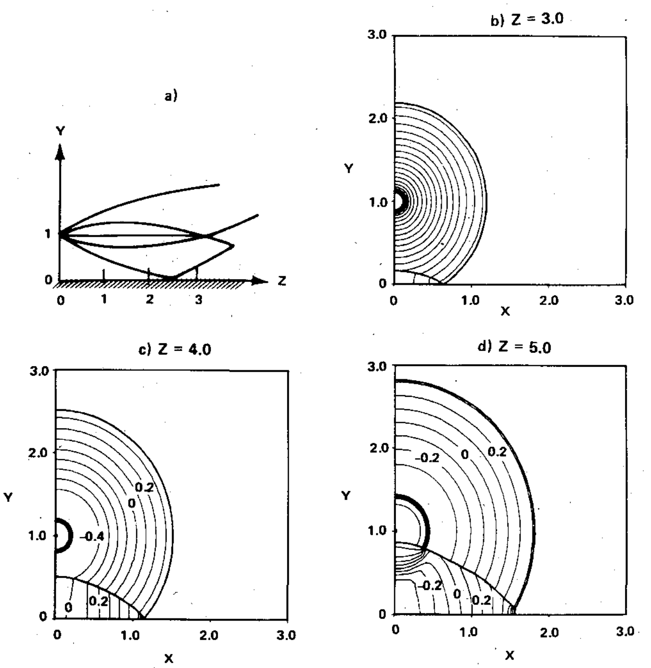

Fig. 12 Cross-sectional shock shape and isobars for a Sears-Haack body; $L/D = 5.5$, $M_\infty = 4$.

performed, and it was found that this was the grid resolution necessary to resolve the flowfield. All computations took about 10 min CPU time on an IBM 370/168 computer. Figure 12 shows the Mach 4 case. A comparison of Figs. 10d and 12d indicates how much more rapidly the Mach disk grows at the lower Mach number.

Conclusions

The complex problem of transition from regular to Mach reflection in three-dimensional steady flow has been studied in detail for the first time. This represents a significant step toward the development of a computational procedure for predicting supersonic store separation/carriage flowfields. It has been found that the detachment criterion for transition is the only one allowing for the determination of the triple point motion at transition. It has also been found that the discontinuous changes in properties at transition are balanced by the derivative of the vertical velocity along the Mach stem. This term plays the same role as the source term in the axisymmetric continuity equation. The flowfield studied here has a number of advantages over the flow situations used in the past for the experimental study of transition from regular to Mach reflection. In particular, this flowfield exhibits a continuous transition, unlike the experiments of Refs. 11 and 12, and if two symmetrical stores are used (instead of a reflection plate), the shock wave/boundary interaction which was present in the experiments of Refs. 13 and 14 is eliminated.

Acknowledgments

The author gratefully expresses his appreciation to Dr. S. Rudman of Grumman's Research and Development Center for his helpful discussions on all aspects of this work. The author also wishes to express his appreciation to N. Talcott of NASA/LRC for supplying his experimental data for comparison. This work was supported in part by the Air Force Office of Scientific Research under Contract F49620-80-C-0044.

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